

# Theory of stripe domains in magnetic shape memory alloys

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The evolution of multivariant patterns in thin plates of magnetic shape memory materials with an applied magnetic field was studied theoretically. A geometrical domain-model is considered composed of straight stripe-like martensite variants with constant internal magnetization (high anisotropy limit) and magnetic domain wall orientation fixed by the twin boundaries. Through integral transforms of the demagnetization energy, the micromagnetic energy is cast into a form convenient for direct numerical evaluation and analytical calculations. The equilibrium geometrical parameters of multivariant patterns with straight and oblique twin boundaries have been derived as functions of the applied field and the material parameters of a plate. It is shown that the oblique multivariant states exist only in plates with thicknesses  $L$  larger than a certain critical value  $L_0$ . In samples with  $L < L_0$  a magnetic-field-driven transformation occurs directly between single variant states.

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## I. INTRODUCTION

In ferromagnetic shape memory alloys like Ni-Mn-Ga Heusler alloys the high magnetocrystalline anisotropy fixes the magnetization along the easy magnetization axis of martensite variants [1].

This “one-to-one correspondence” between magnetic domains and the martensite variants allows to rearrange the crystallographic variants by applying a magnetic field [1, 2]. Hence, magnetic energy determines the evolution of the multivariant states in magnetic fields. The martensitic microstructure can be described by adapting the phenomenological theory of magnetic domains [3, 4]. A detailed understanding of this coupling between microstructure and magnetic domain structure can be achieved for suitably simple geometrical systems.

Here, we develop a theory of multivariant stripes in thin plates. For these stripe structure we derive analytical expressions for demagnetization energies and complete phase diagrams for equilibrium structures.

## II. MICROMAGNETIC ENERGY AND EQUATIONS

As a model we consider multivariant states in a layer of thickness  $L$  with surface normal along  $z$  and infinite extension in  $x$  and  $y$  direction. Commonly observed patterns include lamellar microstructures built from two co-existing single-variant states with oblique or straight interfaces, i.e. the angle between easy-magnetization axis and  $z$  is equal to 0 or  $\pi/4$ , respectively (Fig. 1 a, b).

The orientation of these interfaces with 90-degree magnetic walls are fixed by the crystallography of the twin-boundaries between the martensite variants. Within individual variants stripe domains with 180-degree magnetic domain walls can occur (Fig. 1, c) [5, 6, 7, 8].

The patterns in Fig. 1 a, b on the one hand satisfy the elastic compatibility between the tetragonal variants and, on the other hand, comply with a common property of magnetic domains by avoiding uncompensated magnetostatic charges on the domain boundaries. Here, we assume that internal magnetic charges, that may arise at the twin-boundary [8], are absent or can be neglected as the period of 180-degree domain structures within variants remain small. Further, we assume that the uniaxial anisotropy is much stronger than the applied fields. Thus, deviations of the magnetization from the easy axis within the variants can be neglected. As sketched in Fig. 1 a, the magnetization in the microstructure with oblique twin interfaces can be reduced to a pattern with alternating magnetization  $\mathbf{m} = \pm(M/2)\mathbf{z}$  perpendicular to the plate surface and an effective bias field  $H = H_z + 2\pi M$ . For the pattern with straight interfaces in Fig. 1 b, the alternating magnetization  $\mathbf{m} = \pm(M/\sqrt{2})\mathbf{z}$ , and the bias field is  $H = H_z$ . Thus, the patterns in Fig. 1 are reduced to models of stripe domains with alternating magnetization  $\mathbf{m}$  perpendicular to the surface and *straight* and *oblique* domain walls in a bias field  $H$ . The model with straight domain walls has been investigated in details for magnetic films (see e. g. [4, 9]) and the solutions for the micromagnetic problems can be applied directly for magnetic shape memory films. The model with oblique interfaces is the main subject of our investigations in this paper.

The reduced energy density  $w = W/(2\pi m^2)$  for the oblique stripes can be written

$$w = \frac{2\sqrt{2}p}{\pi} \frac{L_c}{L} - \frac{H}{2\pi m} q + \underbrace{q^2 + \frac{4}{\pi^2 p} \Xi(p, q, \pi/4)}_{w_d}, \quad (1)$$

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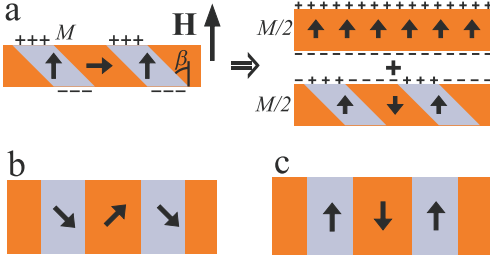


FIG. 1: Multivariant patterns consisting of two different tetragonal martensites, (a)  $\beta = \pi/4$ , (b)  $\beta = 0$ , and stripe domains within one variant (c).

where  $p = 2\pi L/D$ ,  $q = (d_+ - d_-)/D$ , and  $D = d_+ + d_-$  is the period of the domain structure,  $d_{\pm}$  are the widths of domains polarized in the directions parallel (+) and antiparallel (-) to the field. The domain wall energy (the first term in Eq. (1) includes the characteristic length  $L_c = \sigma/(4\pi m^2)$  where  $\sigma$  is the domain wall energy density which describes the balance between the domain wall and stray field energies. The stray field energy  $w_d$  is given by a function  $\Xi(p, q, \pi/4)$  that includes an infinite sum,

$$\Xi(p, q, \beta) = \sum_{n=1}^{\infty} \frac{(1 - (-1)^n \cos(\pi n q))}{n^3} \times [1 - \cos(np \tan \beta) \exp(-np)]. \quad (2)$$

With the help of the integral transformation introduced in [9] the infinite sum in Eq. (3) is transformed into integrals on the interval  $[0, 1]$ . Then, the energy (1) can be written

$$w = 1 - \frac{4p}{\pi^2} \int_0^1 (1-t) \arctan[f(\xi, q)] dt + \frac{p}{\pi} \frac{L_0}{L} - 2hq, \quad (3)$$

where  $h = H/(4\pi m)$ ,  $L_0 = 2\sqrt{2}L_c$ ,  $\xi = pt$ ,

$$f(\xi, q) = 2 \cos^2(\pi q/2) \sinh \xi \sin \xi \times [g(\xi, q) - 2 \cos^2(\pi q/2) (\cosh \xi \cos \xi + \cos \pi q)]^{-1} \text{ and } g(\xi, q) = (\cosh \xi \cos \xi + \cos \pi q)^2 + \sinh^2 \xi \sin^2 \xi.$$

For straight stripes ( $\beta = 0$ ) the integral transformation of sum (3) the following expression gives the system energy [9]

$$w = 1 - \frac{2p}{\pi^2} \int_0^1 (1-t) \ln \left[ 1 + \frac{\cos^2(\pi q/2)}{\sinh^2(pt/2)} \right] dt + \frac{p}{\pi} \frac{L_c}{L} - 2hq. \quad (4)$$

Minimization of Eqs. (3) or (4) with respect to the internal parameters  $p, q$  gives the equilibrium values for the domain sizes ( $d_+, d_-$ ).

### III. RESULTS

#### A. Transition into the single variant states

The difference  $\Delta w = w - w_0$  between the system energy  $w$  and that of the homogeneously magnetized layer,

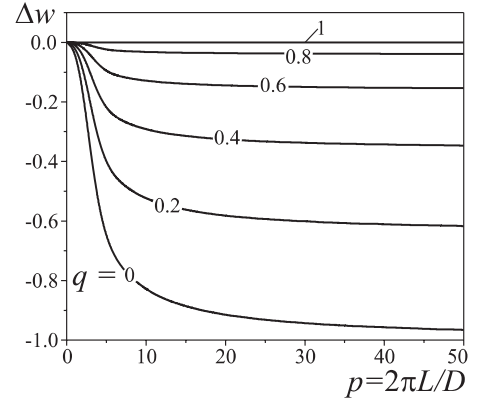


FIG. 2: Energy difference  $\Delta w(p)$  between stripe and single variant equilibrium states as function of the reduced inverse stripe period  $p$  and for different values of the reduced difference  $q$  between widths of up and down domains. The equilibrium energy of the multidomain states is lower than that of the homogeneous state.

$w_0 = 1 - H/(4\pi m)$ , can be written in the following form

$$\Delta w = -\frac{8p}{\pi^2} \cos^2\left(\frac{\pi q}{2}\right) \int_0^1 \frac{(1-t)}{g(\xi, q)} \times \left[ \frac{\xi G(\xi, q)}{\cosh \xi - \cos \xi} - \sinh \xi \sin \xi \frac{\pi(1-q)}{\tan[\pi(1-q)/2]} \right] dt, \quad (5)$$

where  $G(\xi, q) = (\sinh \xi - \sin \xi)(\cos \pi q + \cosh \xi \cos \xi) - \sinh \xi \sin \xi (\sinh \xi + \sin \xi)$ .

The energy difference  $\Delta w(p, q)$ , Eq. (5), is *negative* for all  $p > 0$  and  $0 < q < 1$ , and reaches zero only for  $p = 0$ ,  $q = 1$  (Fig. 2). Similar relations are also true for straight stripes [9]. For both types of systems, stripes with finite widths have *always* lower energy than a single variant state. The stripes transform into a single variant state continuously by unlimited growth of the period  $D$ . Near the transition into the single variant state  $p \rightarrow 0$ , and, thus, the function  $f(\xi)$  in Eq. (3) can be expanded with respect to the small parameter  $\xi$ , and the energy (3) can be evaluated in terms of elementary functions

$$w = 1 - \cos\left(\frac{\pi q}{2}\right) \left[ \frac{2}{\pi u} \frac{L_0}{L} - \frac{2}{\pi^2} \Omega(u) \right] - 2hq, \quad (6)$$

where

$$\Omega(u) = \frac{u}{2} \ln \left( 1 + \frac{4}{u^4} \right) + \ln \left[ \frac{(1 + (u+1)^2)}{(1 + (u-1)^2)} \right] - v(u),$$

$u = 2 \cos(\pi q/2)/p$ , and  $v(u) = 2(1 + u^{-1}) \arctan(1 + u^{-1}) - 2(1 - u^{-1}) \arctan(1 - u^{-1})$ . Minimizing  $w$  from Eq. (6) with respect to  $u$  and  $q$  yields equations for equi-

librium values of  $p$ ,  $q$  and  $u$

$$\begin{aligned} p(u) &= \frac{2}{u} \sqrt{1 - \left(\frac{h}{h^*}\right)^2}, \\ q(u) &= \frac{2}{\pi} \arcsin\left(\frac{h}{h^*}\right), \\ 2\pi L_0 \Upsilon(u) &= L, \end{aligned} \quad (7)$$

where

$$\Upsilon(u) = \left[ u^2 \ln\left(1 + \frac{4}{u^4}\right) + 4 \arctan\left(\frac{u^2}{2}\right) \right]^{-1}, \quad (8)$$

$$\begin{aligned} h^*(u) &= \frac{1}{2\pi} \left[ 2 \arctan\left(\frac{2u}{u^2 - 2}\right) - \right. \\ &\quad \left. u \ln\left(1 + \frac{4}{u^4}\right) - \ln\left(\frac{1 + (u+1)^2}{1 + (u-1)^2}\right) \right]. \end{aligned} \quad (9)$$

Eqs. (7) express the equilibrium values of  $p$  and  $q$  in parametrized form as functions of the applied field  $h$  and the plate thickness  $L$ . According to Eqs. (7) the parameter  $h^*$  from Eq. (9) defines the transition field into the saturated state ( $p = 0$ ,  $q = 1$ ). Note that for  $h = h^*$  the parameter  $u$  transforms into  $u(h^*) = d_-/L$ . This means that the transition into the saturated state takes place by an infinite expansion of the domains with the magnetization parallel to the field, ( $d_+ \rightarrow \infty$ ), while domains with the antiparallel magnetization ( $d_-$ ) keep a finite size. The transition line  $h^*$  plotted as function of  $L_c/L$  in Fig. 3 separates the regions of multivariant and single variant states. The equation  $h^* = 0$  defines the *critical thickness*  $L_0 = 2\sqrt{2}L_c$ . For  $L < L_0$  the oblique domains do not exist, while straight domains (theoretically) exist for any thickness [4, 9] (Fig. 3).

### B. Domain evolution in the magnetic field. Magnetization curves

Typical solutions for domain sizes ( $d_+$ ,  $d_-$ ) are presented in Fig. 4. At low fields, the domain sizes change only slowly. Exponential growth of the size  $d_+$  for the domains with magnetization in field direction sets in close to the saturation field. The minority domain size  $d_-$  gradually decreases in the increasing magnetic field but remains finite at the transition field. The equilibrium magnetization curves in Fig. 5 display different behavior of the magnetization reversal in “thin” ( $L \geq L_c$ ) and “thick” ( $L \gg L_c$ ) plates. The former are characterized by low values of the saturation field  $h^* \ll 1$  and steep magnetization curves. In accordance with the phase theory the latter have linear magnetization curves  $m/m_s = h$  in a broad range of the applied field.

In various aspects, we find that the solutions for the oblique stripes have a *universal* character that may be useful for analysis of experimental data. The solutions

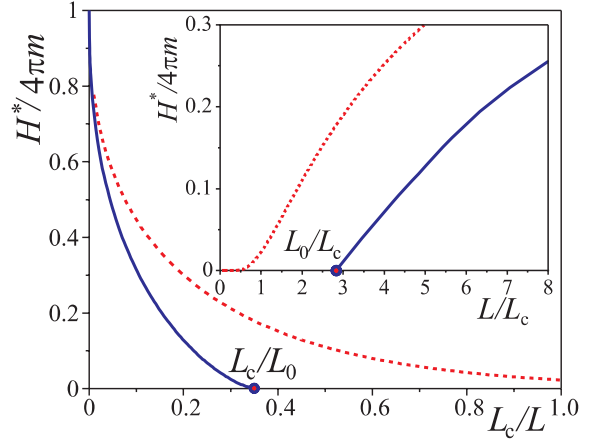


FIG. 3: Phase diagram in terms of the inverse reduced plate thickness  $L_c/L$  and the reduced magnetic field  $H/(4\pi m)$  perpendicular to the plate for oblique (solid) and straight (dashed) stripes. The oblique domains do not exist below the critical thickness  $L_0 = 2\sqrt{2}L_c$ . Inset shows the functions  $H^*(L/L_c)$  close to the critical thickness.

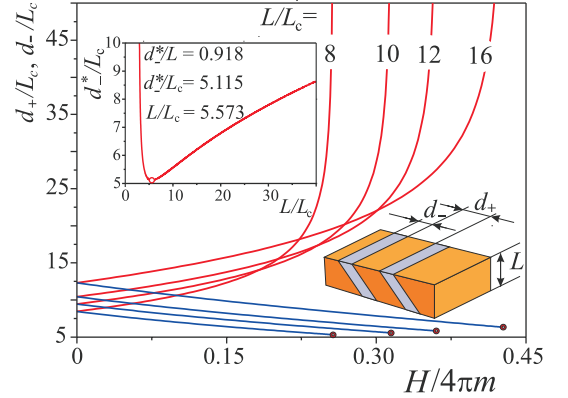


FIG. 4: The equilibrium stripe domain sizes  $d_+/L_c$  (upper set of curves)  $d_-/L_c$  (lower set of curves) as functions of the applied magnetic field for different layer thicknesses. Inset: dependence of the size for the minority domain on layer thickness at the transition.

for  $q$  in Eq. (7) give a universal magnetization curve for systems with large domains  $m/m_s = (2/\pi) \arcsin(h/h^*)$  (Inset in Fig. 5). The solutions for the equilibrium values of  $p$   $(8/\pi) \int_0^1 t \arctan[\sin pt / \sinh pt] dt = L_0/L$  at zero field ( $q = 0$ ) yield a specific dependence of zero-field period  $D_0$  on the layer thickness (Fig. 6).

The equation for  $p$  corresponding to the minimum value of  $D_0$ ,  $\arctan[\sin p / \sinh p] = \int_0^1 t \arctan[\sin pt / \sinh pt] dt$ , includes no material parameters and has the solution  $p_{min} = 2.01$ . This yields  $L^{(min)}/L_c = 4.58$ ,  $D_0^{(min)}/L_c = 14.33$  and the ratio  $D_0^{(min)}/L^{(min)} = 3.13$ . Corresponding values for straight domains are  $\tilde{p}_{min} = 1.595$ ,  $\tilde{L}^{(min)}/L_c = 3.8178$ ,  $\tilde{D}_0^{(min)}/L_c = 14.5739$ ,  $\tilde{D}_0^{(min)}/\tilde{L}^{(min)} = 3.8176$  [10]. The minority domain size,  $d_-^*/L_c(L/L_c)$ ,

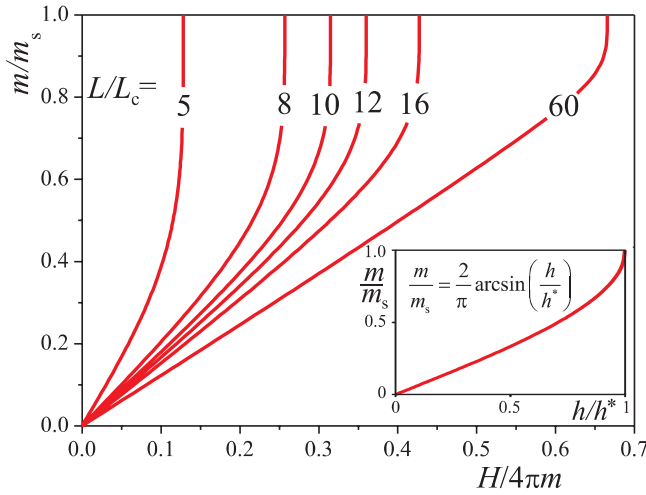


FIG. 5: Magnetization curves for different values of the reduced thickness  $L/L_c$ . Inset shows the “universal” character of the magnetization curves for large domains.

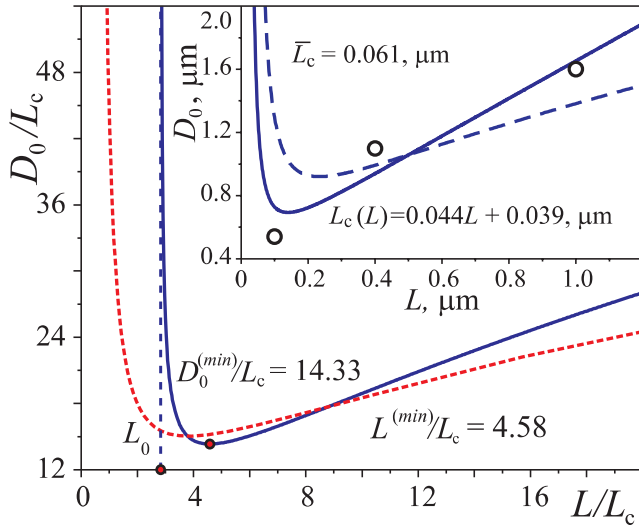


FIG. 6: The equilibrium period  $D_0/L_c$  as a function of the plate thickness  $L/L_c$  at zero field (solid line) in comparison with a corresponding line (dashed) for straight stripes. Inset shows  $D(L)$  functions for a thin layer of  $\text{Ni}_{51.4}\text{Mn}_{28.3}\text{Ga}-20.3$  investigated in Ref. [7] corresponding to the straight stripe structure Fig. 1, b. The dashed curve is plotted for the averaged characteristic length  $\bar{L}_c$ , the solid line incorporates effects connected with the indicated variation of  $L_c$  with  $L$ .

at the transition field has a similar minimum (Inset in Fig. 4). The equation for  $x = \min(d_-^*/L)$   $4 \arctan(x^2/2) - x^2 \ln(1 + 4/x^4) = 0$  has the solution  $x = 0.918$ , and yields the following values of the parameters in this point:  $L/L_c = 5.573$ ,  $d_-^*/L_c = 5.115$ .

The point  $D_0^{(min)}$  marks the transition region between a functional dependence  $D_0 \propto \sqrt{L}$  characteristic for thick plates ( $D_0 \ll L$ ) (Kittel law) to an exponential growth of the period with a decreasing plate thickness,  $D_0 \propto \exp(-\pi L_c/L)$  [9] in thin layers ( $L \geq L_c$ ). This transitional region has been reached in thin polycrystalline films of  $\text{Ni}_{51.4}\text{Mn}_{28.3}\text{Ga}-20.3$  with easy-axes dominantly at 45-degree from the layer normal direction [7]. Inset in Fig. 6 shows the calculated functions  $D_0(L)$  with material parameter  $L_c$  adjusted to the experimental data of Ref. [7].

#### IV. CONCLUSIONS

The present theory enables rigorous calculations of equilibrium structures in thin ferromagnetic martensite plates with two variants. It is clear that real magnetization processes in magnetic shape-memory processes will generally be hysteretic. However, the equilibrium states could be reached, e.g., free-standing films or thin free single-crystal platelets with very mobile twin-boundaries. The analytical expressions for the energy of a stripe system Eqs. (1)–(energyST) are applicable also for non-equilibrium states. We find that oblique stripe structures are possible only above a certain critical thickness, while straight stripe structures have been known to exist for arbitrary dimensions of layers with perpendicular magnetic anisotropy. The theoretical method can be extended to treat twinned martensite plates with other angle between easy magnetization and layer normal direction.

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